**MATHEMATICAL MODEL**

***Indices and Index:***

|  |  |
| --- | --- |
| *i, j* | *index of Job* |
| *o* | *index of operation* |
| *m* | *index of machine* |
| *J* | *number of jobs* |
| *O* | *number of operations per each job* |
| *M* | *number of machines* |

***Set***

|  |  |
| --- | --- |
| *SJ* | *Set of jobs* |
| *SO* | *Set of operations* |
| *SM* | *Set of machines* |

***Parameter:***

|  |  |
| --- | --- |
|  | *Processing time of operation o job j* |
|  | *Binary, precedence assignment of operation o of job j on machine m, it means that the operation o of job j must be processed on machine m* |
|  | *A very large number* |

***Decision variable:***

|  |  |
| --- | --- |
|  | *Starting time of operation o of job j* |
|  | *Completion time of operation o of job j* |
|  | *Binary, =1 if job i is processed after job j at operation o on machine m* |
|  | *Binary, =1 if job i and job j at operation o is assigned on machine m* |
|  | *Completion time of job j* |
|  | *The makespan of the schedule* |

***Objective:***

|  |  |
| --- | --- |
| *Minimize* |  |

***Subject to:***

|  |  |  |
| --- | --- | --- |
|  |  | *(1)* |
|  |  | *(2)* |
|  |  | *(3)* |
|  |  | *(4)* |
|  |  | *(5)* |
|  |  | *(6)* |
|  |  | *(7)* |
|  |  | *(8)* |
|  |  | *(9)* |
|  |  | *(10)* |
|  |  | *(11)* |